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Modeling orientation fields of fingerprints with rational complex functions

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Abstract

In this paper, a novel model is proposed for the orientation field of fingerprints, which can be expressed as the argument of a rational complex function. It is suitable for all types of fingerprints. Experimental results show that it performs much better than the previous works.

Keywords: Fingerprint; Orientation field; Rational complex function

1. Introduction

Among various biometric techniques, automatic fingerprint recognition is most popular and reliable for automatic personal identification. Fingerprint is the pattern of ridges and valleys on the surface of a fingertip. Its *orientation field* is defined as the local orientation of the ridge-valley structures and the singular points (including cores and deltas) can be viewed as points where the orientation field is discontinuous. Most classical fingerprint recognition algorithms take the minutiae (ridge endings and bifurcations) and the singular points, including their coordinate and direction, as the distinctive features to represent the fingerprint in the matching process. But obviously, this kind of representation cannot provide enough information for large-scale fingerprint identification tasks [1].

Orientation field describes one of the basic structures of a fingerprint. It has been widely used for minutiae extraction and fingerprint classification, but rarely utilized into the matching process. In this paper, we focus on the modeling of orientation field. Our purpose is to represent orientation field in a complete and concise form so that it can be accurately reconstructed with a small number of coefficients. It can be used to improve the estimation of orientation field; therefore it will benefit the extraction of minutiae for conventional fingerprint identification algorithms. More importantly, the coefficients of the orientation field model can be saved for the usage in the matching step.

Sherlock and Monro [2] had proposed a so-called zero-pole model for orientation field based on singular points, which takes core as zero and delta as pole in complex plane. The influence of a core z_c, is $\frac{1}{2}\arg(z - z_c)$ for a point, z, and that of a delta z_d, is $-\frac{1}{2}\arg(z - z_d)$. The orientation at z is the sum of the influence of all cores and deltas. Vizcaya and Gerhardt [3] had made an improvement using a piecewise linear approximation model around singular points to adjust the zero and pole's behavior. The neighborhood of each singular point is divided into eight regions and the influence of the singular point is assumed to change linearly in each region. In these two models, the influence of a singular point is the same for any point on the same central line, so they are effective only near the singular points and lack of accuracy far from singular points. Furthermore, both the two models cannot deal with "plain arch" fingerprints which have no singular points. In a word, these two models could not meet the need of real applications.

Here we propose a novel model for the orientation field of fingerprint using a rational complex function, which can accurately describe orientation fields for all types of finger-

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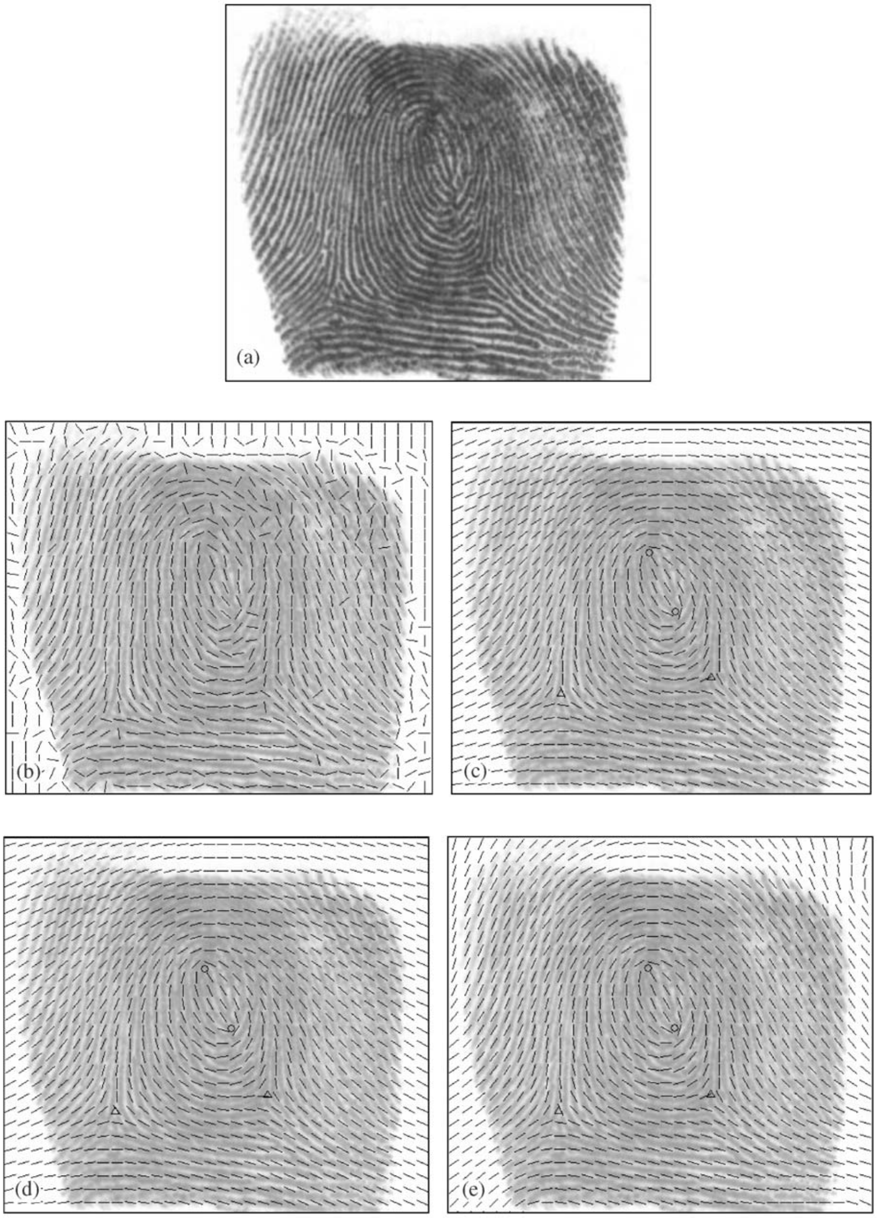

Fig. 1. Comparative result of the orientation field constructed by using three models: (a) original fingerprint image, (b) coarse orientation field estimated by using the algorithm in Ref. [4], (c) zero-pole model, (d) piecewise linear model, and (e) our model.

prints at regions whether near or far from singular points. Consequently, it is feasible to utilize the model for the improvement of orientation field's estimation and further application to fingerprint matching stage.

2. Rational complex model for the orientation field

Denote the image plane as a complex space, C. For any $z \in C$, the value of fingerprints' orientation, $\theta(z)$, is defined within $[0, \pi)$, so it can be regarded as half the argument of a complex number, i.e. $\theta(z) = \frac{1}{2}\arg U(z)$. As we know, the orientation pattern of a fingerprint is quite smooth and continuous except at the singular points (including cores and deltas), so a rational complex function may be utilized here to represent the function, $U(z)$, in which the known cores and deltas act as zeros of the numerator and the denominator, respectively. Thus, the model for the orientation field can be defined as

$$\phi(z) = \frac{1}{2}\arg\left[\frac{f(z)}{g(z)} \cdot \frac{P(z)}{Q(z)}\right], \qquad (1)$$

where $P(z) = \prod_{i=1}^{s_0}(z - z_c^i)$, $Q(z) = \prod_{j=1}^{s_1}(z - z_d^j)$, $\{z_c^i\}_{1 \leqslant i < s_0}$ and $\{z_d^j\}_{1 \leqslant j < s_1}$ are the cores and deltas of the fingerprint in the known region. The zeros of $f(z)$ and $g(z)$ should be outside the known region. Obviously, the zeros of $f(z)$, $g(z)$, $P(z)$ and $Q(z)$ will define the model.

Obviously, the zero-pole model proposed in Ref. [2] can be seen as a special example of our model, where $f(z)$ and $g(z)$ are both set to a constant, such as 1. It also should be noted that our model is suitable for all types of fingerprints, even for "plain arch" fingerprints (without singular points).

3. Computation of the model's parameters

From the mathematical theorem about complex variables, we know a rational function can be approximated by polynomial function in a closed region. So, to simplify the computation of the rational complex model, the model can be written as

$$\phi(z) = \frac{1}{2}\arg\left[f(z) \cdot \frac{P(z)}{Q(z)}\right]. \qquad (2)$$

We want to find a function, $f(z)$, to minimize the difference between $\{\phi(z)\}$ and the original orientation field, $\{\theta(z)\}$. Denote $\psi(z) = \frac{1}{2}\arg[P(z)/Q(z)]$ and $\omega(z) = \frac{1}{2}\arg[f(z)]$, and now what we need to do is to compute $f(z)$ by minimizing the difference between $\{\omega(z)\}$ and $\{\theta(z) - \psi(z)\}$.

It is unsuitable for us to directly compute this minimum. A solution to this problem is mapping the orientation field to a continuous complex function by using

$$U(z) = \cos 2[\theta(z) - \psi(z)] + \mathrm{i}\sin 2[\theta(z) - \psi(z)]. \qquad (3)$$

Then, instead of computing the minimal difference between $\{\omega(z)\}$ and $\{\theta(z) - \psi(z)\}$, we will compute the function, $f(z)$, by minimizing $\sum_z |f(z) - U(z)|^2$. Since $\theta(z)$, $P(z)$ and $Q(z)$ (corresponding to the original orientation field, cores and deltas) are known when we deal with a fingerprint image, it is easy to solve this problem by using least-squares error principle.

When we choose $f(z)$ from the set of polynomials with order less than n, only $n + 1$ parameters need to be computed and saved (which is much less than the model of Ref. [3]). In our experiments, we set n as 6. Due to the global approximation, our model has a robust performance against noise.

4. Experimental results

The experiments are carried on more than 100 inked fingerprints and live-scanned fingerprints. These fingerprints are of different types: loop, whorl, twin loop, and plain arch without singular point. They also vary in different qualities.

Three orientation models, zero-pole model [2], piecewise linear model [3] and our model, are evaluated on the database. All of them use the same algorithm for singular points extraction and orientation estimation algorithms proposed in Ref. [4]. On all these fingerprint images, our model has a rather satisfying performance. The average approximation error of our model is about 6°, which is much better than using the other two models (14° and 11°, respectively). The experimental results also show that our model, though based on the coarse orientation field, can reconstruct the orientation field smoothly and accurately.

Fig. 1 gives an example for comparison, where (a) is the original fingerprint, (b) is the coarse orientation field estimated by using the algorithm in Ref. [4], (c)–(e) are the reconstructed orientation fields, respectively, by using the zero-pole model, piecewise linear model and our model. The reconstructed orientation field is shown as unit vectors upon the original fingerprint. As shown, the zero-pole model and piecewise linear model perform badly in the place far from the singular points, which can be easily observed in the top-left and the top-right part in (c) and (d). Instead, our model can describe the orientation of the whole fingerprint image precisely.

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References

[1] S. Pankantic, S. Prabhakarb, A.K. Jain, On the individuality of fingerprints, IEEE Trans. Pattern Anal. Mach. Intell. 24 (8) (2002) 1010–1025.
[2] B. Sherlock, D. Monro, A model for interpreting fingerprint topology, Pattern Recognition 26 (7) (1993) 1047–1095.
[3] P. Vizcaya, L. Gerhardt, A nonlinear orientation model for global description of fingerprints, Pattern Recognition 29 (7) (1996) 1221–1231.
[4] A.M. Bazen, S.H. Gerez, Systematic methods for the computation of the directional fields and singular points of fingerprints, IEEE Trans. Pattern Anal. Mach. Intell. 24 (7) (2002) 905–919.