A Novel Model for Orientation Field of Fingerprints

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Abstract

As a global feature of fingerprint, orientation field is very important to automatic fingerprint identification system (AFIS). Establishing an accurate and concise model for orientation field will not only improve the performance of orientation estimation, but also make it feasible to apply orientation information into the matching process. In this paper, such a novel model for orientation field of fingerprint is proposed. We use a polynomial model to approximate the orientation field globally and a point-charge model at each singular point to improve the approximation locally. These two models are combined together by a weight function. Experimental results are provided to illustrate this combination model is more accurate and robust to noise compared with the previous works. Its applications are discussed at the end.

1 Introduction

Among various biometric techniques, automatic fingerprint identification system (AFIS) is most popular and reliable for automatic personal identification. During the last years its performance has reached a high level. However, it is still not satisfying for a large database or fingerprints with poor-quality [1].

Fingerprint is the pattern of ridges and valleys on the surface of a fingertip. The ridges are black and the valleys are white. Its *orientation field* is defined as the local orientation the ridge-valley structures. The *minutiae* are defined as ridge endings and bifurcations. The *singular points* can be viewed as points where the orientation field is discontinuous, which can be classified into two types: *core* and *delta*. Most classical AFIS algorithms [1, 2, 3, 4, 5] take the minutiae and the singular points, including their coordinate and direction, as the distinctive features to represent the fingerprint in the matching process. But obviously, this kind of representation does not utilize all available features in the fingerprints and therefore cannot provide enough information for large-scale fingerprint identification tasks [6]. Develop a more complete representation for fingerprints will surely result in much better performance.

As a global feature, orientation field describes one of the basic structures of a fingerprint. Moreover, the variation of orientation field is of low frequency so that it is robust to various noises. It has been widely used for minutiae extraction and fingerprint classification, but rarely utilized into the matching process. In this paper, we focus on the modeling of orientation field. Our purpose is to represent orientation field in a complete and concise form so that it can be accurately reconstructed with several coefficients. This work's significance lies in the following three aspects: (1) It can be used to improve the estimation of orientation field, especially for poor-quality fingerprints; therefore it will benefit the extraction of minutiae. (2) More importantly, orientation field can be utilized for fingerprint matching after modeling, thus a much better identification performance could be expected. (3) It is possible to establish a complete representation for the fingerprint by combining the orientation model with some other information such as minutiae and ridge density map [7].

Sherlock [8] had proposed a so-called zero-pole model for orientation field based on singular points, which takes core as zero and delta as pole in complex plane. The influence of a core z_c, is $\frac{1}{2}arg(z - z_c)$ for point z, and that of a delta z_d, is $-\frac{1}{2}arg(z - z_d)$. The orientation at z, is the sum of the influence of all cores and deltas. It is simple and effective, but lack of accuracy, because obviously many fingerprints that have nearly the same singular points yet differ in detail. Vizcaya [9] had made an improvement using a piecewise linear approximation model around singular points to adjust the zero and pole's behavior. First, the neighborhood of each singular point is uniformly divided into eight regions and the influence of the singular point is assumed to change linearly in each region. An optimization implemented by gradient-descend is then carried on to get piecewise linear function. It is more adaptable to real fingerprint indeed, but not quite smooth. For poor-quality fingerprint, the gradient-descend optimization is not always convergent. Furthermore, these two models cannot deal with fingerprint which contains no singular point, such as plain arch classified by Henry [8, 10]. Since they do not consider the distance from singular points, the influence of a singular point is the same to any point on the same central line, either near or far from the singular point. This will cause serious error in the modeling of the regions far from singular points.

Here we propose a combination model for orientation matrix. Since orientation of fingerprint is quite smooth and

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continuous except at singular points, we apply a polynomial model to approximate the global orientation field. At each singular point, a point-charge model similar with zero-pole model is used to describe the local region. Then, these two models are combined smoothly together through a weight function. Features of the combination model are as below: (1) It can accurately represent orientation field at regions whether near or far from singular points. (2) Global approximation makes it robust against noise. (3) It has a concise representation, which guarantees a low storage cost for its application into fingerprint identification.

2 The Model of Orientation Field

As the value of fingerprints' orientation is often defined on $[0, \pi)$, it seems that this representation has an intrinsic discontinuity. So it is unsuitable to model the orientation field directly. A solution is mapping the orientation field to a continuous complex plane[11][12]. Denote $\{\theta(x, y)\}$ as the orientation field. The mapping is defined as:

$$U = R + iI = \cos(2\theta) + i\sin(2\theta) \tag{1}$$

where R and I denote the real part and image part of the unit-length complex, U, respectively. Obviously, $R(x, y)$ and $I(x, y)$ are continuous with x, y in those regions. The above mapping is a one-to-one transformation and $\theta(x, y)$ can be easily reconstructed.

Now, we can equivalently model the orientation field in two ways: one is to model $U(x, y)$ in complex domain; the other is to model $R(x, y)$ and $I(x, y)$ respectively in real domain. The latter is employed in this paper and the former one will be touched in our further research. Specially, a bivariate polynomial model is chosen for $R(x, y)$ and $I(x, y)$ (denoted by PR, PI) respectively, which can be formulated as:

$$\begin{pmatrix} 1 & x & \cdots & x^n \end{pmatrix} \begin{bmatrix} p_{00} & p_{01} & \cdots & p_{0n} \\ p_{10} & p_{11} & \cdots & p_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n0} & p_{n1} & \cdots & p_{nn} \end{bmatrix} \begin{pmatrix} 1 \\ y \\ \vdots \\ y^n \end{pmatrix}$$

where the order n can be determined ahead.

Near the singular points, it is difficult to be modeled with polynomial functions. A model named Point-Charge is added at each singular point. Compared with the model provided in [8], Point-Charge uses different quantity of electricity to describe the neighborhood of each singular point, instead of same influence at all singular points. And for a certain singular point, its influence at point (x, y) varies with the distance between the point and the singular point. The influence of a standard (vertical) core at point (x, y) , is defined as:

$$PC_{core} = (H_1, H_2) = \begin{cases} \left(\frac{y-y_0}{r}, \frac{x-x_0}{r}\right) Q & r \le R \\ (0, 0) & r > R \end{cases} \tag{2}$$

where (x_0, y_0) is this core's position, Q is the quantity of electricity, R denotes the radius of its effective region, and $r = \sqrt{(x - x_0)^2 + (y - y_0)^2}$. And that of a standard delta is:

$$PC_{delta} = (H_1, H_2) = \begin{cases} \left(\frac{y-y_0}{r}, -\frac{x-x_0}{r}\right) Q & r \le R \\ (0, 0) & r > R \end{cases} \tag{3}$$

In real fingerprint, the ridge pattern near the singular points usually has a rotation angle compared with the standard one. If the rotation angle from standard position is $\phi(\phi \in (-\pi, \pi])$, a transformation can be made as:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} x - x_0 \\ y - y_0 \end{pmatrix} \tag{4}$$

Then, the Point-Charge model can be modified by taking x' and y' instead of x and y, for cores in (2) and deltas in (3), respectively.

To combine the polynomial model (PR, PI) with Point-Charge smoothly, a weight function is defined. For Point-Charge, its weight at (x, y) is defined as:

$$\alpha_{PC}^{(k)}(x, y) = 1 - \frac{r^{(k)}(x, y)}{R^{(k)}} \tag{5}$$

where $(x_0^{(k)}, y_0^{(k)})$ is the coordinate of the k-th singular point, $R^{(k)}$ is its effective radius, and $r^{(k)}(x, y)$ is set as $\min\left(\sqrt{(x - x_0^{(k)})^2 + (y - y_0^{(k)})^2}, R^{(k)}\right)$.

For polynomial model, its weight at (x, y) is:

$$\alpha_{PM}(x, y) = \max\left(1 - \sum_{k=1}^{K} \alpha_{PC}^{(k)}, 0\right) \tag{6}$$

where K is the number of singular points. The weight function guarantees that for each point, its orientation follows the polynomial model if it is far from the singular points and follows the Point-Charge if it is near one of the singular points.

Finally, the combination model for the whole fingerprint's orientation field can be formulated as:

$$\begin{pmatrix} R(x, y) \\ I(x, y) \end{pmatrix} = \alpha_{PM} \cdot \begin{pmatrix} PR \\ PI \end{pmatrix} + \sum_{k=1}^{K} \alpha_{PC}^{(k)} \cdot \begin{pmatrix} H_1^{(k)} \\ H_2^{(k)} \end{pmatrix} \tag{7}$$

3 Implementation Scheme

3.1 Original Orientation Field Computation

There are essentially two ways to compute the orientation field of fingerprint: filter-bank based approaches [15] and gradient-based approaches [4, 13, 14]. The filter-bank based approaches are more resistant to noise than the latter, but it is discrete valued (depending on the number of filters) and computationally expensive. So we adopt the latter one [13] to compute the original orientation field O . The coherence denoted by W , or reliability of O can also be obtained along with it.

We also need to acquire the position and type of singular points. Many approaches have been proposed for singular point extraction. Most of them are based on Poincare index [3, 4, 14, 15]. In this paper, we first detect the singular points by Poincare index; then an optimization on a local 5×5 window is carried on to get the precise position.

3.2 Polynomial Approximation

Given order n , the polynomial model is first formulated into a linear model by taking each factor $x^i y^j$ as an independent variable. Then, weighted least squares (WLS) is applied here to implement the approximation. The reliability, $W(x,y)$, is used as the weight factor at point (x,y). The reason we use weight factor here is that this method can efficiently decrease the influence of inaccurate orientation estimation. As pointed above, the reliability $W(x,y)$ can indicate how well the orientation is fit for the real ridge. The more the reliability $W(x,y)$ is, the more influence the point should have.

From experiments, we find that 4-order($n = 4$) polynomial is enough for global approximation.

3.3 Point-Charge Model at Singular Point

Point-Charge Models at singular points are obtained in two steps. First, two parameters are estimated for each singular point: ϕ-the rotation angle and R-the effective radius. Second, charges of singular points are estimated by optimization.

As ϕ describes the average orientation of the ridge pattern around the singular points, we set its value as the polynomial model's output at the singular points. Note that $\phi \in (-\pi, \pi]$. For cores, we need to tell whether it is upward or downward. This is trivial in fact. We solve this problem by matching the core with an upward core and a downward core template, which are generated from standard Point-Charge Model. Therefore, for the singular point at (x_0, y_0), its rotation angle ϕ is estimated as:

Delta and Upward Core:

$$\phi(x_0, y_0) = \begin{cases} \frac{\pi}{2} - \alpha & \alpha = \frac{1}{2}\tan^{-1}\frac{PI(x_0,y_0)}{PR(x_0,y_0)} \geq 0 \\ -\frac{\pi}{2} - \alpha & \text{otherwise} \end{cases} \tag{8}$$

Downward Core:

$$\phi_{downward}(x_0, y_0) = \phi_{upward}(x_0, y_0) + \pi \tag{9}$$

For the convenience of computation, we use a same R for each singular point. The value can be determined empirically.

After that, we need to estimate the charges for singular points. Since our primitive purpose is to minimize the approximate error, the objective function for the singular points can be represented as:

$$\min J = \sum_{\Omega} \left[(R(x,y) - \cos 2O)^2 + (I(x,y) - \sin 2O)^2 \right]$$

where O is the original orientation field and Ω is the effective region for the point-charge model. For each singular point, its effective region is a small circle with radius R. Ω is the union set of all these small circles. The variables in the above optimization problem are the charges of singular points, $\{Q_1, Q_2, \cdots, Q_K\}$. They can be computed by solving the following equations as:

$$\partial J / \partial Q_k = 0, \quad k = 1, 2, \cdots, K \tag{10}$$

4 Experimental Results

The experiment is carried on two sets of fingerprints. The first set (Set 1) contains 60 fingerprint images captured with a live-scanner, whose size is 512×320(pixels). The second set (Set 2) is a sample database from NIST Special Database 14 [16] that contains 40 fingerprint images. The images' size is 480×512(pixels). The fingerprints in these two sets vary in different qualities and types.

Three orientation models are evaluated on the database: zero-pole model [8], piecewise linear model [9] and our combination model. All of them use the same algorithm for singular points extraction and orientation estimation. The combination model employs 4-order polynomial. As mentioned above, the orientation field extracted by Gabor filter-bank (when the number of filters is large enough) is more reliable than the original one based on gradient computing. So, the approximation error of a fingerprint is defined as mean absolute error (MAE) on all points between the orientation field reconstructed by the model and the orientation field extracted by Gabor filter-bank [15] (64-filters). Then, by averaging the total approximation error on all fingerprints in the database, the error of each model can be gained along with its standard deviation. The results summarized in Table 1 shows that our combination model leads to a notable reduction both in the mean error and the standard deviation than the other two models.

From observation, it can also be concluded that our combination model has a satisfying performance, which is much better than the other two models. Some results of our combination model are presented in Fig.1. Among them there are various fingerprint types: loop, whorl, and plain arch without singular point (the other two models can not deal with plain arch fingerprints). The reconstructed orientation matrices are shown as unit vectors upon the original fingerprint. As shown, the result is rather accurate and robust to noise. Fig.2 gives another example for comparison, where (a) is the original fingerprint (a poor-quality loop), (b)(c)(d) are the reconstructed orientation fields, respectively by zero-pole, piecewise linear and our combination model. Result shows that: Zero-pole can only roughly describe the real orientation without accuracy. Piecewise linear model does better near the singular points, but it fails in the place far from them, such as in the left and the right bottom part in Fig.2(c). Moreover, it is not very smooth. Instead, the combination model can describe the orientation of the whole fingerprint image smoothly and precisely, whether the region is near or far from the singular points. It also works well against noise.

Assuming n-order polynomial is applied for a certain fingerprint with 4 singular points (i.e. 2 cores and 2 deltas), the total number of coefficients (which need to be saved) is $2(n+1)^2+4$(2 matrices PR, PI, and 4 charges for modeling singular points). As n is chosen as 4 and implemented with Matlab6.1 and C on Pentium III 500Hz PC, it costs about 1.5 seconds for modeling and 100 bytes for storage on average, which is quite suitable for real application.

5 Conclusion and Discussion

To sum up, a combination model for orientation field of fingerprints is proposed in this paper. Results show this model is more accurate and reliable than the previous work. Moreover, it can deal with fingerprint without singular points. It can also be implemented efficiently and suitable for on-line processing.

Our further work will lie in two aspects. First, our combination model deals with the smoothly continuous ridge pattern and singular points separately, and then combines them together. As we mentioned above, modeling U in complex domain is an alternative way where we can take core as zero and delta as pole. A rational function in complex domain may be employed for U, which will be more universal and concise.

Another aspect of further work is the application of this model. First, as pointed previously, minutiae points, orientation map and ridge density map can completely describe a fingerprint image. We can use the orientation model to compress, restore or synthesize the fingerprint images. Secondly, it is possible to develop new method for fingerprint identification based on the ridge orientation model, in which the coefficients of orientation model can be used for fingerprint matching.

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Table 1. Mean orientation error of three models(°)

	Zero-Pole	Piecewise Linear	Combination Model
Mean	14.32	10.64	5.58
Standard Deviation	5.47	4.15	2.42

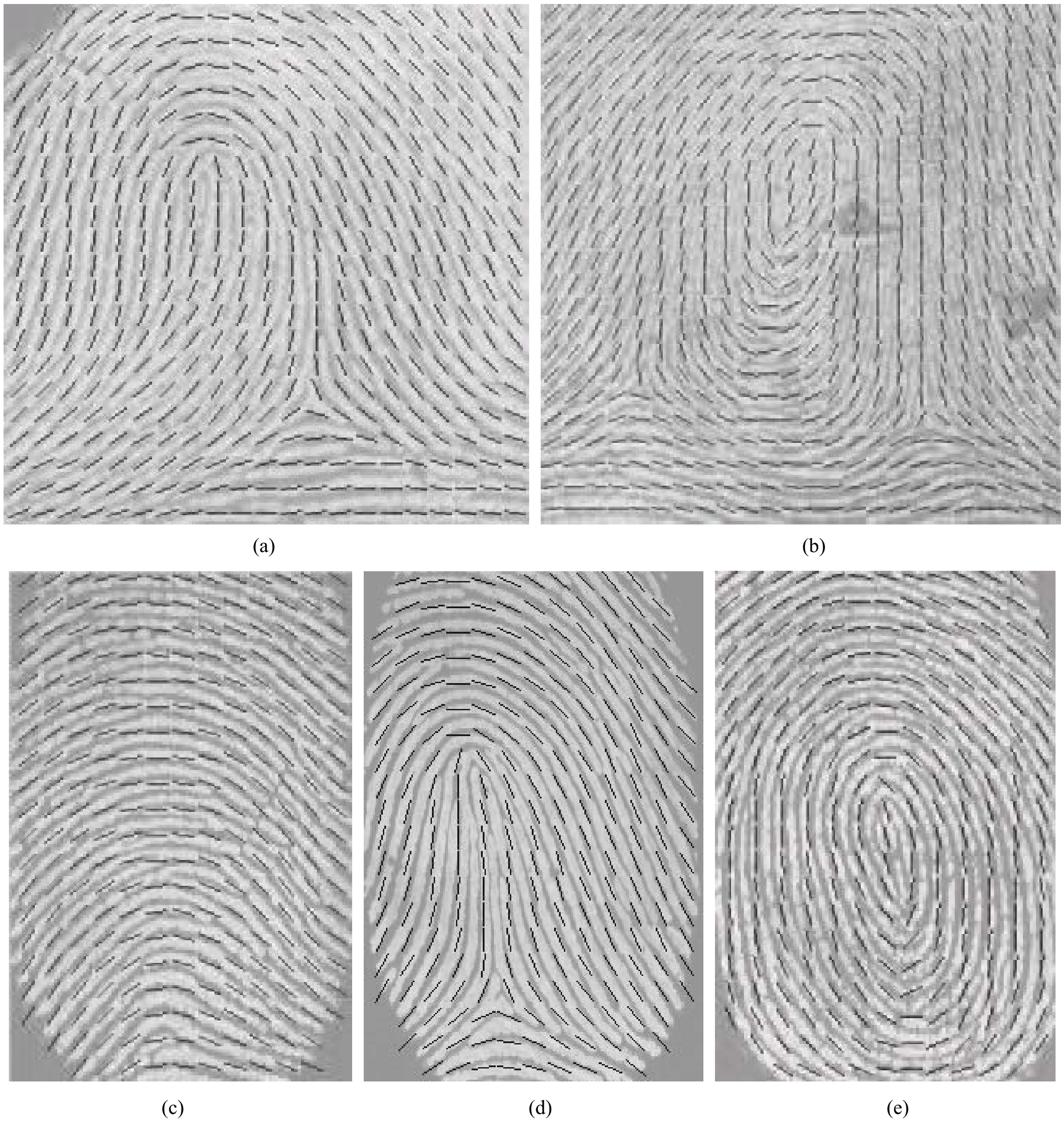

Figure 1. Reconstructed orientation field using our combination model. (a) and (b) are from Set 2, in which (a) is a loop and (b) is a whorl; (c), (d) and (e) are from Set 1, in which (c) is a plain arch without singular point, (d) is a loop, and (e) is a whorl. In contrast, zero-pole model and piecewise linear model cannot deal with the plain arch as in (c).

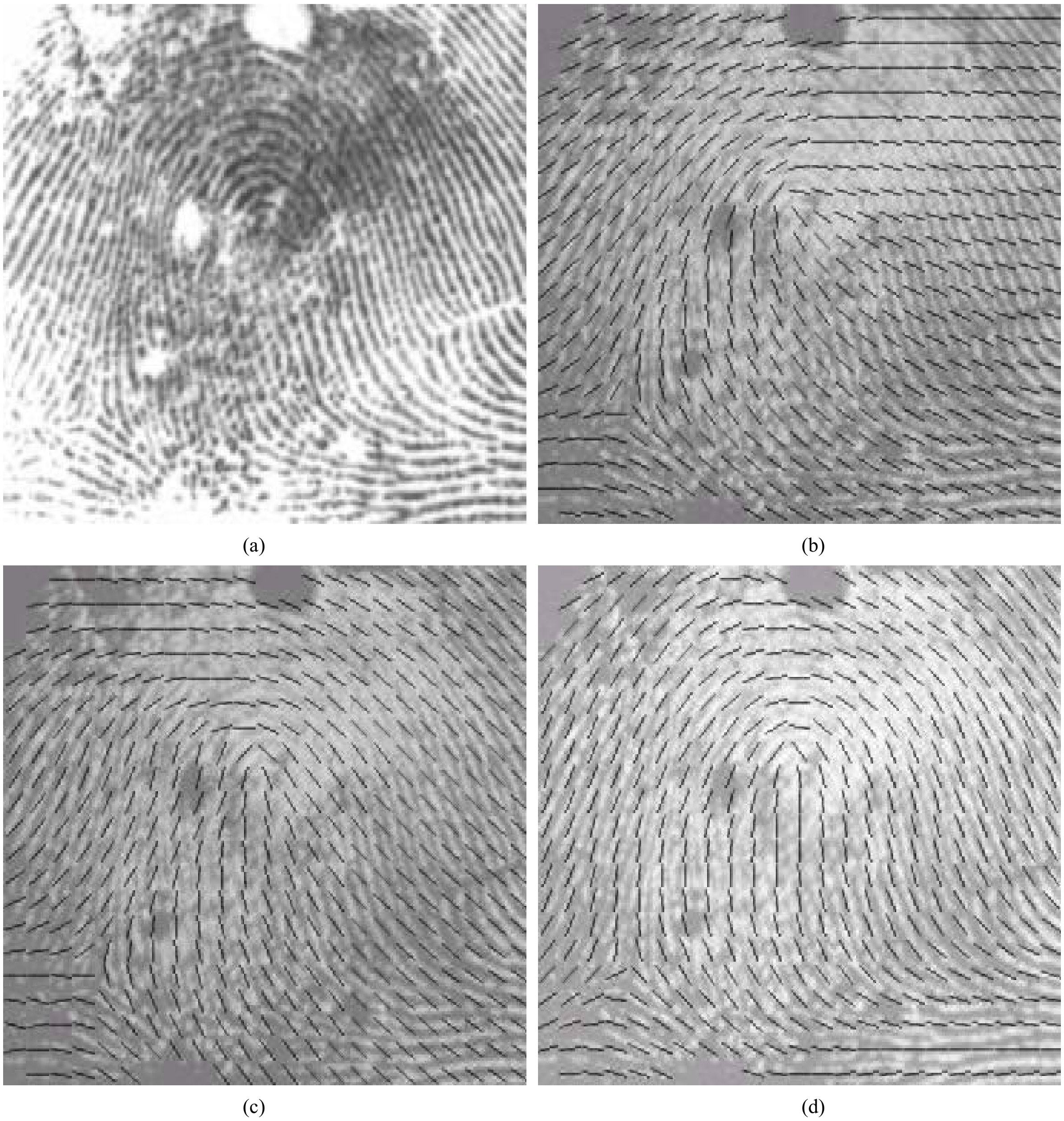

Figure 2. Reconstructed orientation of a poor-quality loop from Set 2 by three models: (a) Original fingerprint, (b) zero-pole, (c) piecewise linear model, and (d) the combination model. Zero-pole roughly describes the orientation. Piecewise linear model fails in the places far from the singular points, such as the left and the right bottom part in (c) compared with (d).